



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

ROOT LOCUS TECHNIQUE

SKETCHING OF ROOT LOCUS-II

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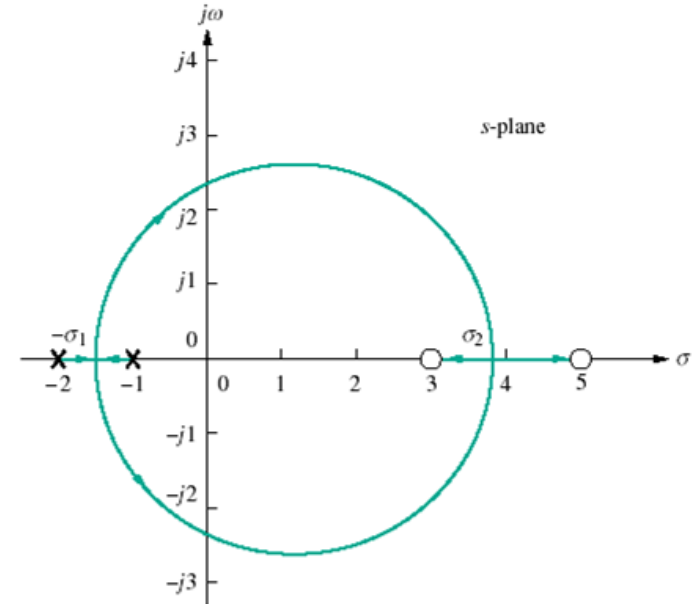
REFINING THE SKETCH OF ROOT LOCUS (RL)

1. *Real-axis BREAKAWAY & BREAK-IN points*
2. The $j\omega$ –axis crossings (transition from stability to instability)
3. Angles of Departure and Arrival



REAL-AXIS BREAKAWAY & BREAK-IN POINTS

- The point where the locus leaves the real-axis is called the BREAKAWAY point
- The point where the locus returns to the real-axis is called the BREAK-IN point
- At the breakaway or break-in point, the branches of the root locus form an angle of $(180/n)$ deg. with the real-axis. n = no. of closed loop poles arriving at or departing from the single breakaway or break-in point on the real axis.



REAL-AXIS BREAKAWAY & BREAK-IN POINTS

On the root locus & on the real – axis,

$$KG(s)H(s) = -1 = KG(\sigma)H(\sigma)$$

$$KG(\sigma)H(\sigma) = -1 \Rightarrow K = -\frac{1}{G(\sigma)H(\sigma)}$$

$$\frac{dK(\sigma)}{d\sigma} = 0.$$

BA and BI points satisfy the following relationship

$$\sum_1^m \frac{1}{\sigma + z_i} = \sum_1^n \frac{1}{\sigma + p_i}$$

Where z_i & p_i are the negative of the zero and pole values, respectively of $G(s)H(s)$.

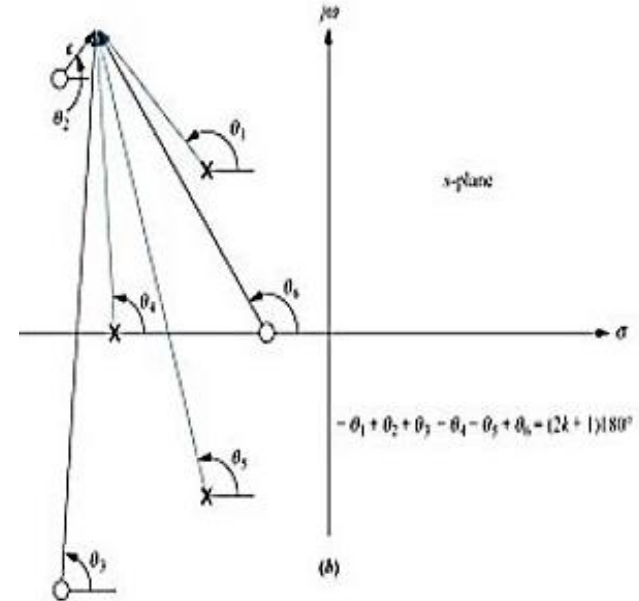
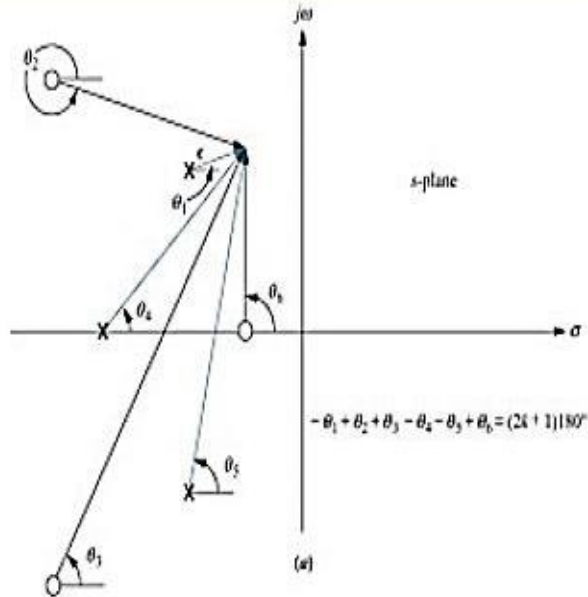
Solving for σ yields the BA/BI points.

THE $j\omega$ -AXIS CROSSINGS

- The $j\omega$ –axis crossings points on the RL that separates the stable operation from the unstable operation
- To find the $j\omega$ –axis crossings, we can use the Routh-Hurwitz criterion as follows:
- Forcing a row of zeros in the Routh table will yield the gain; going back one row to the even polynomial equation and solving for the roots yields the frequency at the imaginary axis crossing
- The sum of angles from the finite open-loop poles and zeros must add to $(2k+1)180$ degree. Thus we can search $j\omega$ –axis until we find the point that meets this angle condition
- The value of ω at the axis crossing yields the frequency of oscillation, while the gain at the $j\omega$ –axis crossings yields, the maximum (or minimum) +ve gain for system stability

ANGLES OF DEPARTURE AND ARRIVAL

- To calculate the RL departure angle from the complex poles and the arrival angle to the complex zeros



REFERENCE BOOKS

- Norman S. Nise, Control Systems Engineering, Wiley, 2013
- Katsuhiko Ogata, Modern Control Engineering, Prentice Hall, 2010.

